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ERROR STATISTICS FROM PREDICTING UTI-UTC AND POLE POSITION.(U)

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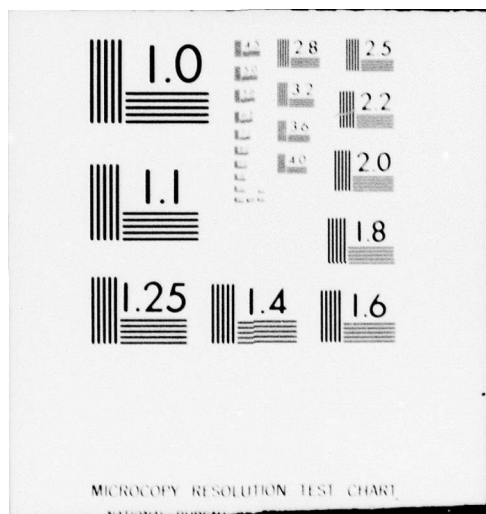
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# FOREWORD

Precisely determining the ephemerides of high-altitude satellites, such as the Global Positioning System (GPS) satellites, requires accurately predicted values of UT1-UTC and pole position for orienting the earth's crust in inertial space. This report presents the results of a study that compared several methods of predicting UT1-UTC and pole position. Acknowledgement is gratefully made to Mike Harkins for his assistance and technical advice.

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## CONTENTS

	<u>Page</u>
FOREWORD . . . . .	i
LIST OF TABLES . . . . .	iv
INTRODUCTION . . . . .	1
UT1-UTC . . . . .	2
POLE POSITION . . . . .	9
SUMMARY . . . . .	13
REFERENCES . . . . .	14
DISTRIBUTION	



# LIST OF TABLES

<u>Table</u>		<u>Page</u>
1	UTl-UTC Prediction Errors (msec) From First-Degree Polynomia and Two-Point (10-Day) Fit Spans . . . . .	3
2	UTl-UTC Prediction Errors (msec) From a First-Degree Polynomial With Fit Spans of Four to 24 Points (20 to 120 Days) . . . . .	4
3	UTl-UTC Prediction Errors (msec) From a Second-Degree Polynomial With Fit Spans of Four to 24 Points (20 to 120 Days) . . . . .	4
4	UTl-UTC Prediction Errors (msec) From a Third-Degree Polynomial With Fit Spans of Four to 73 Points (20 to 365 Days) . . . . .	5
5	UTl-UTC Prediction Errors (msec) From a Fourth-Degree Polynomial With Fit Spans of Four to 73 Points (20 to 365 Days) . . . . .	5
6	Power Spectrum Analysis . . . . .	6
7	UTl-UTC Prediction Errors (msec) From Equation (1) Using Different Values For n . . . . .	7
8	UTl-UTC Prediction Errors (msec) From Equation (1) Comparing n = 6 to n = 4 . . . . .	7
9	Comparison of Different Principle Periods for UTl-UTC Errors (1974) Used in Equation (1) With n = 6 . . . . .	8
10	Pole Position Prediction Errors (sec of arc) From Past Method With a 40-Point Fit Span . . . . .	10
11	Pole Position Prediction Errors (sec of arc) for 1975 From Equation (3) . . . . .	10
12	Pole Position Prediction Errors (sec of arc) for 1976 From Equation (3) . . . . .	11
13	Pole Position Prediction Errors (sec of arc) From Equation (3) Plus Semiannual Terms; Data From 1975 . . . . .	11
14	Pole Position Prediction Errors (sec of arc) From Equation (3) Plus Semiannual Terms; Data From 1976 . . . . .	12

## INTRODUCTION

For the Global Positioning System (GPS) satellites, precise ephemerides will have to be determined by a least-squares fit to ground tracking data. The parameters of this fit will then be used to calculate 7- to 14-day predicted ephemerides. To do these fits and predictions, the earth's crust, or more particularly the location of the tracking sites which are located on the crust, must be accurately oriented in inertial space at all times. Since the GPS satellites' ephemerides have longer and more accurate prediction requirements than previous satellites, more accurate values of UT1-UTC and pole position are needed to rotate from the earth-fixed reference frame into the inertial frame.

UT1-UTC is the time difference between a nonuniform time representing the true angular rotation of the earth and a uniform time standard referenced to atomic clocks. It has a yearly drift of about 1 sec per year. It also has smaller seasonal and shorter-term variations along with other unpredictable fluctuations.

Past programs, incorporating UT1-UTC values that were given every seventh day, basically used the two most recent values to fit a straight line, and then used this straight line to predict values.<sup>1</sup> However, the UT1-UTC values fluctuate from a straight line; therefore, more accurate predicted values may be obtained by fitting a first-degree or higher-order polynomial with more than two data points. Since there are seasonal and other periodic fluctuations in the UT1-UTC data, a trigonometric series with a 365-day principle period may also produce more accurate predictions. Equation (1) details the calculations for such a trigonometric series.

$$UT1-UTC = A + Bt + \sum_{m=1}^n C_m - \sin \frac{2\pi mt}{365} + D_m \cos \frac{2\pi mt}{365} \quad (1)$$

where

t = time in days

n = number of cosine terms

A, B, C<sub>m</sub>, D<sub>m</sub> = linear coefficients found in fit

Pole position is defined by the coordinates of the instantaneous axis of rotation referenced to the CIO (Convention International Origin) standard pole. The pole has periodic motions that have a principle period of about 435 days, the Chandler Period, and a period of about one year. At present, X values (measured in the direction of Greenwich) and Y values (measured perpendicular to X) for the pole position are found by doing a least-squares fit using 200 days of data to determine the linear coefficients, as shown in Equation (2).<sup>1</sup>



$$F(t) = A + B_1 \sin \frac{2\pi t}{P_1} + C_1 \cos \frac{2\pi t}{P_1} \quad (2)$$

where

$t$  = time in days

$A$ ,  $B_1$ , and  $C_1$  = linear coefficients

$P_1$  = Chandler Period = 420 days

The fits for  $X$  and  $Y$  are carried out separately, and the results are used to predict new values of pole position.

Improved pole positions may be obtained by adding another term, representing the yearly fluctuations in position, to the trigonometric series in Equation (2):

$$F(t) = A + B_1 \sin \frac{2\pi t}{P_1} + C_1 \cos \frac{2\pi t}{P_1} + B_2 \sin \frac{2\pi t}{P_2} + C_2 \cos \frac{2\pi t}{P_2} \quad (3)$$

where

$P_2$  = yearly period = 365 days

UT1-UTC and pole position data from 1973 through 1976 was used for the fit and prediction experiments to test how well some of these methods would perform. The remainder of this report discusses these tests.

The 5-day data for these tests was obtained from the annual reports of the Bureau International De L'Heure (BIH).<sup>2</sup>

#### UT1-UTC

Raw values of UT1-UTC obtained from Table 6C of the BIH annual reports were used to make fits and check the accuracy of predicted values. Each year of data was divided into 23 spans, each 20 days long, a new span starting every 15 days. For each span, a least-squares fit for the linear parameters of each equation considered was made using a selected time span of data immediately preceding the prediction span. Using the fitted parameters in the considered equation, predicted values of UT1-UTC were computed at 5, 10, 15, and 20 days into the span associated with each fit. Since the BIH

data is all given once every five days, each 5-, 10-, 15-, and 20-day prediction could be compared to a BIH value. For each year, the root-mean-squares (rms) of the prediction errors for the 5-day predictions were then computed and the maximum error for that year was found. This same computation was done for 10-, 15-, and 20-day predictions.

Table 1 shows the prediction error statistics that the present system (a first-degree polynomial fit to two points) would create.

Table 2 gives the prediction error statistics obtained from using a first-degree polynomial and four (20 days) to 24 (120 days) points in the least-squares fit span. Likewise, Tables 3, 4, and 5 show prediction error statistics for second-, third-, and fourth-degree polynomials, respectively, with different fit spans used to determine coefficients. The smallest errors seem to be obtained using a first-degree polynomial with four data points in the least-squares fit.

Table 6 is a power spectrum analysis which shows that some of the small variations are seasonal and periodic, with at least three major periods: 360, 180, and 90 days. To fit this effect, a trigonometric series was added to the first-degree polynomial (see Equation (1)) in an attempt to improve UT1-UTC prediction accuracy.

Table 7 shows prediction error statistics obtained from using one year of data to do a least-squares fit for the linear coefficients in Equation (1). The number of cosine terms in the series (n) was varied from 1 to 15. The best error statistics were obtained from  $n = 4$  to  $n = 6$ . Table 8 further compares  $n = 4$  and  $n = 6$  for data from 1973 and 1976.

The power spectrum analysis in Table 6 indicates that n must be at least as large as 4 to fit the major frequencies in the UT1-UTC data. But, it seems that if n is larger than 6 there will be too many terms for a good prediction.

Table 9 shows the effect on error statistics of changing the principle period from 365 days.

Table 1. UT1-UTC Prediction Errors (msec\*) From First-Degree Polynomials and Two-Point (10-Day) Fit Spans

Year	Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
	rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
1974	4.28	10.60	6.74	14.00	10.33	-20.20	13.15	-26.50
1975	2.98	-6.3	4.42	-10.70	6.66	-14.70	8.66	-20.20

\* 0.001 sec  $\approx$  50 cm at the surface of the earth.

Table 2. UT1-UTC Prediction Errors (msec) From a First-Degree Polynomial

Fit			Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
Number of Points	Days	Year	rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
4	20	1974	3.28	- 7.55	4.67	10.29	6.26	11.28	7.74	15.87
		1975	1.99	- 3.75	2.97	- 6.41	4.03	8.38	5.61	9.47
8	40	1974	3.68	- 8.32	4.88	-10.48	6.41	-12.35	8.12	18.03
		1975	2.54	- 4.66	3.83	- 7.74	5.50	11.09	6.77	12.71
12	60	1974	5.13	-10.31	6.82	-14.32	8.86	-18.16	10.91	-24.82
		1975	4.05	6.69	5.86	-10.19	8.00	-14.57	9.37	-16.75
16	80	1974	7.35	-16.11	9.43	-21.29	11.66	-22.87	13.96	-26.96
		1975	5.86	-10.58	8.08	15.55	10.54	-19.99	11.99	-22.05
20	100	1974	9.61	-18.23	11.98	-23.74	14.26	-25.66	16.79	-31.23
		1975	8.01	-14.58	10.49	20.43	13.16	-25.24	14.67	-26.57
24	120	1974	11.88	-22.39	14.32	-26.30	16.51	-28.80	19.17	-35.71
		1975	10.22	-18.19	12.85	25.37	15.61	-29.36	17.16	31.94

Table 3. UT1-UTC Prediction Errors (msec) From a Second-Degree Polynomial

Fit			Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
Number of Points	Days	Year	rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
4	20	1974	5.79	-15.9	11.70	-28.45	21.10	-45.72	31.64	-65.03
		1975	3.68	6.93	6.74	14.17	11.32	22.96	17.11	35.20
8	40	1974	3.93	- 7.92	6.18	-11.13	8.62	-15.49	11.4	22.02
		1975	2.35	- 4.22	3.51	- 7.01	4.29	7.75	6.85	-12.61
12	60	1974	3.38	- 7.03	4.68	-11.19	6.24	-13.12	8.15	-18.72
		1975	2.50	- 4.40	3.63	- 7.17	4.78	9.70	7.14	14.81
16	80	1974	3.82	- 8.01	5.33	- 9.69	7.70	-12.47	10.06	-18.99
		1975	3.09	- 6.26	4.29	-10.04	5.87	11.75	8.25	-16.97
20	100	1974	5.00	- 9.69	6.82	13.33	9.61	17.38	12.08	-22.99
		1975	4.08	- 8.94	5.70	-13.83	7.75	-15.63	10.27	-19.95
24	120	1974	6.49	-12.63	8.88	16.16	12.08	20.99	14.86	-26.74
		1975	5.29	-11.27	7.37	-16.95	9.88	-19.62	12.51	-22.39



Table 4. UT1-UTC Prediction Errors (msec) From a Third-Degree Polynomial

Fit Number of Points	Days	Year	Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
			rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
4	20	1974	11.08	-27.50	35.56	-97.1	80.38	-220.1	150.63	-119.7
		1975	9.11	-20.00	29.43	-60.4	67.87	-138.8	127.71	262.6
8	40	1974	5.70	-13.74	12.73	-24.72	24.58	44.14	39.96	70.76
		1975	3.09	-6.59	6.53	-14.65	12.18	27.31	20.09	44.15
12	60	1974	4.13	-9.45	7.38	-15.15	11.41	-23.46	16.87	35.01
		1975	2.39	4.59	3.72	6.72	5.43	12.45	8.74	18.77
16	80	1974	3.51	-7.94	5.35	-13.27	6.75	-16.86	9.53	-24.71
		1975	2.53	4.37	4.04	7.69	5.52	10.55	8.64	16.22
20	100	1974	3.60	-6.39	5.32	-9.38	7.40	14.05	10.62	20.02
		1975	2.65	4.68	3.99	7.01	5.44	10.11	8.51	15.10
24	120	1974	4.22	8.64	6.02	13.67	8.84	19.30	12.38	-24.47
		1975	3.27	5.96	4.69	8.86	6.52	12.72	9.86	19.18
36	180	1974	7.50	13.84	10.76	21.56	15.23	28.69	19.61	34.35
		1975	6.35	-12.50	8.79	-18.78	12.06	-22.14	16.44	-31.98
73	365	1974	15.72	26.91	19.09	30.82	22.27	36.86	25.36	43.84

Table 5. UT1-UTC Prediction Errors (msec) From a Fourth-Degree Polynomial

Fit Number of Points	Days	Year	Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
			rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
8	40	1974	9.93	22.17	29.47	61.17	67.20	131.7	128.2	241.5
		1975	5.86	11.61	15.73	31.34	35.51	65.35	67.63	-123.1
12	60	1974	6.41	-16.84	14.9	-33.90	30.03	-60.88	51.2	-98.56
		1975	2.78	-4.81	5.88	12.55	10.67	25.94	18.77	44.14
16	80	1974	4.34	-10.02	8.40	17.32	14.80	30.17	22.86	48.44
		1975	2.54	4.88	4.74	-7.69	8.08	-15.84	13.45	-25.11
20	100	1974	3.56	-69.59	6.50	13.49	9.80	22.63	14.70	35.40
		1975	2.49	5.28	4.14	-9.44	6.44	15.95	10.36	24.89
24	120	1974	3.88	-7.89	6.64	-13.47	9.45	17.87	14.15	27.86
		1975	2.42	-4.82	4.13	-10.25	5.98	-13.81	9.66	-24.52
28	140	1974	3.94	7.81	6.42	12.55	9.36	17.92	14.05	-24.52
		1975	2.71	-5.98	4.39	-8.08	6.39	-15.55	10.10	-18.66
36	180	1974	5.21	-10.25	7.73	-13.95	11.46	-22.84	16.57	-30.25
		1975	3.64	8.04	5.62	11.34	8.13	16.34	12.32	22.59
73	365	1974	17.56	-30.51	23.54	-39.84	30.30	-46.69	37.34	-57.41
		1975	14.70	-28.27	19.99	-36.53	25.65	-46.53	31.57	-57.33

Table 6. Power Spectrum Analysis

K	Period <sup>a</sup>	Omega <sup>b</sup>	Spectrum Analysis Coefficients		Regression Power	Percent Power
			A	B		
1	364.0000	0.017261	8.9953E-03	-9.4324E-03	8.5048E-05	41.974450
2	182.0000	0.034523	1.2500E-02	-7.1446E-03	1.0288E-04	50.777446
3	121.3333	0.051784	-1.2624E-03	2.7095E-04	8.2168E-07	0.405534
4	91.0000	0.069046	2.7170E-03	6.2323E-04	3.8260E-06	1.888281
5	72.8000	0.086307	-1.0499E-03	5.0833E-04	6.7482E-07	0.333051
6	60.6667	0.103569	5.7245E-04	1.2127E-06	1.6124E-07	0.079580
7	52.0000	0.120830	-5.4955E-04	-5.4963E-04	3.0108E-07	0.148596
8	45.5000	0.138092	6.6233E-04	2.8004E-04	2.5505E-07	0.125879
9	40.4444	0.155353	-7.5433E-04	4.9901E-04	4.0808E-07	0.201405
10	36.4000	0.172615	-6.0503E-04	3.3795E-04	2.3914E-07	0.118025
11	33.0909	0.189876	-7.2282E-04	-2.5044E-05	2.5751E-07	0.127094
12	30.3333	0.207138	6.3857E-04	6.6714E-04	4.2437E-07	0.209442
13	28.0000	0.224399	2.2470E-04	-7.2824E-04	2.9490E-07	0.145546
14	26.0000	0.241661	5.6011E-04	4.7582E-04	2.6773E-07	0.132138
15	24.2667	0.258922	-4.1753E-04	2.5824E-04	1.2049E-07	0.059465
16	22.7500	0.276184	7.1193E-04	1.6117E-04	2.6227E-07	0.129439
17	21.4118	0.293445	-1.1293E-04	-2.8320E-04	4.6683E-08	0.023040
18	20.2222	0.310707	3.4686E-04	2.3037E-04	8.5603E-08	0.042248
19	19.1579	0.327968	-7.2147E-04	4.1132E-05	2.5810E-07	0.127382
20	18.2000	0.345230	2.7265E-04	9.9344E-05	4.1470E-08	0.020467
21	17.3333	0.362491	1.0824E-04	1.2892E-04	1.4065E-08	0.006941
22	16.5455	0.379753	3.8374E-04	3.6539E-05	7.3358E-08	0.036205
23	15.8261	0.397014	-7.9478E-04	6.6525E-05	3.1537E-07	0.155649
24	15.1667	0.414276	7.4689E-05	1.7639E-04	1.8326E-08	0.009045
25	14.5600	0.431537	-4.4975E-05	3.1218E-04	5.0310E-08	0.024830
26	14.0000	0.448799	2.5223E-04	6.7883E-05	3.3690E-08	0.016628
27	13.4815	0.466060	8.8975E-05	-1.8705E-04	2.1740E-08	0.010729
28	13.0000	0.483322	2.7742E-05	-4.9165E-05	1.6146E-09	0.000797
29	12.5517	0.500583	-4.6465E-04	-1.3336E-04	1.1571E-07	0.057109
30	12.1333	0.517845	6.3632E-04	2.0552E-04	2.2163E-07	0.109383
31	11.7419	0.535106	-1.3604E-04	-4.6923E-04	1.1827E-07	0.058371
32	11.3750	0.552368	-9.0599E-05	2.9918E-04	4.8999E-08	0.024183
33	11.0303	0.569629	1.5300E-04	7.8584E-06	1.1863E-08	0.005855
34	10.7059	0.586891	1.9246E-04	2.2767E-04	4.3586E-08	0.021512
35	10.4000	0.604152	-2.7344E-04	-5.0426E-07	3.9321E-08	0.019406
36	10.1111	0.621414	1.6221E-04	-2.0493E-04	3.3101E-08	0.016337
37	9.8378	0.638675	2.9976E-05	1.7890E-04	1.8972E-08	0.009363

<sup>a</sup> Period = Principle Period (364 days)/K

<sup>b</sup> Omega =  $\frac{2\pi}{\text{Period}}$



Table 7. UT1-UTC Prediction Errors (msec) From Equation (1) Using Different Values For n

n	Year	Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
		rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
1	1974	17.44	-28.51	19.95	-32.87	21.83	-36.60	24.15	-38.56
	1975	12.58	-19.87	14.90	-23.73	16.84	-26.54	17.37	-26.70
2	1974	4.68	-10.11	6.00	-12.48	6.82	-17.08	7.80	-16.59
	1975	2.51	- 5.56	3.24	- 6.23	4.27	- 8.72	4.23	- 9.32
3	1974	4.09	- 9.08	5.43	-11.20	6.17	-14.96	7.55	-14.72
	1975	2.29	- 4.05	2.98	- 6.13	3.83	- 5.82	4.05	- 7.57
4	1974	3.42	- 8.20	4.62	-10.22	5.31	-11.12	6.89	-11.30
	1975	1.74	3.20	2.09	4.35	2.65	5.32	3.22	7.56
5	1974	3.22	- 8.44	4.54	-10.78	5.33	-10.93	6.86	-12.22
	1975	2.03	- 3.63	2.48	- 5.07	3.09	- 6.22	3.89	- 8.80
6	1974	3.03	- 7.36	4.41	- 9.35	5.29	-10.30	6.94	-10.53
	1975	2.11	- 4.20	2.59	- 5.75	3.12	- 6.91	4.03	- 9.43
7	1974	2.85	- 6.90	4.29	- 9.52	5.37	- 9.70	7.01	-10.86
	1975	2.11	4.04	2.56	- 6.17	3.19	- 7.68	4.06	- 9.68
8	1974	3.23	- 8.87	4.57	-11.32	5.66	-10.80	7.39	-12.48
10	1974	3.60	- 9.20	5.43	-12.16	7.11	-11.89	8.68	-16.48
15	1974	3.42	- 9.80	5.30	-11.91	7.06	-12.56	8.59	-13.97

Table 8. UT1-UTC Prediction Errors (msec) From Equation (1) Comparing n = 6 to n = 4

n	Year	Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
		rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
6	1973	1.93	- 3.36	3.11	- 6.97	2.69	6.75	3.17	5.53
	4	2.20	- 4.69	3.35	7.30	3.12	8.35	3.40	7.17
6	1974	3.03	- 7.36	4.41	- 9.35	5.29	-10.30	6.94	-10.53
	4	3.42	- 8.20	4.62	-10.22	5.31	-11.12	6.89	-11.30
6	1975	2.11	- 4.20	2.59	- 5.75	3.12	- 6.91	4.03	- 9.43
	4	1.74	- 3.20	2.09	- 4.35	2.65	- 5.32	3.22	- 7.56
6	1976	3.04	-10.80	4.49	-13.4	6.23	-15.4	6.75	-16.6
	4	3.10	- 9.23	4.28	-11.3	5.95	-13.0	6.54	-14.3

Table 9. Comparison of Different Principle Periods for UT1-UTC Errors (1974)  
Used in Equation (1) With  $n = 6$

Principle Period (Days)	Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
	rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
425	4.20	-8.90	6.25	-13.11	8.44	-15.43	11.23	-23.03
405	3.73	-9.13	5.41	-11.83	7.14	-14.97	9.35	-22.02
385	3.61	-8.98	5.01	-11.49	6.41	-11.62	8.15	-16.49
365	3.03	-7.36	4.41	-9.35	5.29	-10.30	6.94	-10.53
345	3.45	-8.67	5.15	-11.68	6.21	-13.24	8.10	-16.86
325	3.92	-8.87	5.88	-11.64	7.58	-12.51	10.29	-19.06

## POLE POSITION

Smoothed values for X and Y were obtained from Table 7 of the BIH annual reports. Similar to the UT1-UTC test, each year of data for X and Y was divided into 18 spans, each 20 days long, a new span starting every 20 days. For each span, a least-squares fit for the linear coefficients was made using the data immediately before the 20-day span as the fit span. Predicted values were then calculated for 5, 10, 15, and 20 days and compared to the BIH data. Rms and maximum prediction errors were calculated as they were for UT1-UTC predictions.

Table 10 shows the prediction error statistics in the X and Y components of pole position that come from using the old method of computing pole position predictions; that is, fitting Equation (2) with 40 points (200 days) of data.

Table 11 shows prediction error statistics in the X and Y components of pole position that come from using Equation (3) and data from 1974 and 1975. All predictions are in 1975. Three different values (445, 435, and 425 days) were used for the Chandler Period, and three different fit spans were used for each Chandler Period. The prediction errors seem to change very little with changes in Chandler Period. These data fit spans would probably have to be long enough so that each would be a multiple of one year and also a multiple of the Chandler Period in order to be used to help determine the Chandler Period.

Table 12 shows prediction error statistics in the X and Y components of pole position for data from 1976. These errors seem to have about the same sensitivity to the Chandler Period as the errors in Table 11.

Tables 13 and 14 are the same as Tables 11 and 12, respectively, except another cosine term with a period of 182 days was added to the trigonometric series of Equation (3). This term improves the prediction errors for the Y component of pole position, but the errors in the X component get worse. Since this 182-day term improves Y predictions but not X, there must be some variation present in the Y pole data that has a period of about 182 days. Further experiments would be of interest to determine if the same situation exists for International Polar Monitoring Service (IPMS) pole positions, which are based on a different treatment of seasonal effects, or Doppler pole positions, which are subject to smaller tropospheric refraction effects and no star catalog errors.

Table 10. Pole Position Prediction Errors (sec of arc\*) From Past Method  
With a 40-Point Fit Span

Year	Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
	rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
X POLE								
1975	0.0134	-0.0270	0.0162	-0.0314	0.0189	-0.0366	0.0218	-0.0406
1976	0.0093	-0.0185	0.0123	0.0239	0.0122	-0.0251	0.0136	-0.0269
Y POLE								
1975	0.0143	0.0232	0.0176	-0.0290	0.0212	-0.0344	0.0248	-0.0406
1976	0.0190	0.0297	0.0230	-0.0371	0.0273	-0.0433	0.0315	-0.0504

\* 0.01 sec of arc  $\approx$  30 cm at the surface of the earth.

Table 11. Pole Position Prediction Errors (sec of arc) For 1975 From  
Equation (3)

Chandler Period (Days)	Number of Points in fit	Pole Component	Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
			rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
445	89	X	0.0090	-0.0200	0.0101	-0.0223	0.0110	-0.0245	0.0118	-0.0253
		Y	0.0188	0.0305	0.0217	0.0359	0.0247	0.0444	0.0276	0.0502
	87	X	0.0098	-0.0212	0.0110	-0.0236	0.0121	-0.0268	0.0130	-0.0296
		Y	0.0186	0.0290	0.0217	0.0358	0.0249	0.0442	0.0281	0.0500
	85	X	0.0104	-0.0219	0.0118	-0.0260	0.0131	-0.0302	0.0141	-0.0335
		Y	0.0184	0.0280	0.0217	0.0347	0.0251	0.0428	0.0284	0.0485
435	89	X	0.0090	-0.0198	0.0100	-0.0219	0.0110	-0.0240	0.0116	-0.0251
		Y	0.0186	0.0293	0.0215	0.0344	0.0243	0.0424	0.0271	0.0479
	87	X	0.0098	-0.0209	0.0110	-0.0232	0.0120	-0.0267	0.0128	-0.0294
		Y	0.0185	0.0279	0.0216	0.0344	0.0247	0.0424	0.0278	0.0479
	85	X	0.0104	-0.0217	0.0118	-0.0258	0.0130	-0.0300	0.0140	-0.0331
		Y	0.0182	0.0270	0.0214	0.0333	0.0248	0.0412	0.0281	0.0466
425	89	X	0.0091	-0.0196	0.0101	-0.0216	0.0110	-0.0234	0.0116	-0.0250
		Y	0.0184	0.0281	0.0212	0.0326	0.0240	0.0402	0.0267	0.0453
	87	X	0.0098	0.0217	0.0110	0.0229	0.0121	0.265	0.0129	0.0291
		Y	0.0184	0.0266	0.0214	0.0328	0.0244	0.0404	0.0274	0.0456
	85	X	0.0104	-0.0217	0.0118	-0.257	0.0131	-0.0297	0.0141	-0.0327
		Y	0.0181	0.0258	0.0213	0.0319	0.0245	0.394	0.0278	0.0445



Table 12. Pole Position Prediction Errors (sec of arc) For 1976 From Equation (3)

Chandler Period (Days)	Number of Points in fit	Pole Component	Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
			rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
445	89	X	0.0082	0.0177	0.0096	0.0211	0.0114	0.0235	0.0130	0.0253
		Y	0.0222	0.0312	0.0256	0.0358	0.0290	0.0414	0.0321	0.0470
	87	X	0.0074	0.0164	0.0089	0.0196	0.0107	0.0218	0.0123	0.0251
		Y	0.0227	0.0326	0.0264	0.0374	0.0301	0.0432	0.0336	0.0490
	85	X	0.0067	0.0153	0.0082	0.0183	0.0098	0.0204	0.0114	0.0238
		Y	0.0230	0.0330	0.0270	-0.0382	0.0311	0.0438	0.0350	0.0496
435	89	X	0.0086	0.0183	0.0100	0.0219	0.0120	0.0246	0.0137	0.0264
		Y	0.0221	0.0314	0.0254	0.0361	0.0288	0.0416	0.0318	0.0472
	87	X	0.0078	0.0170	0.0092	0.0204	0.0112	0.0228	0.0129	0.0258
		Y	0.0226	0.0328	0.0262	0.0376	0.0299	0.0434	0.0333	0.0493
	85	X	0.0070	0.0159	0.0085	0.0191	0.0103	0.0214	0.0119	0.0245
		Y	0.0228	0.0332	0.0268	0.0381	0.0308	0.0440	0.0347	0.0500
425	89	X	0.0091	0.0190	0.0105	0.0228	0.0127	0.0257	0.0146	0.0277
		Y	0.0220	0.0320	0.0253	0.0368	0.0286	0.0418	0.0316	0.0473
	87	X	0.0082	0.0177	0.0097	0.0212	0.0119	0.0239	0.0137	0.0266
		Y	0.0225	0.0329	0.0260	0.0378	0.0297	0.0437	0.0331	0.0496
	85	X	0.0074	0.0166	0.0089	0.0199	0.0109	0.0224	0.0126	0.0253
		Y	0.0227	0.0334	0.0266	0.0384	0.0306	-0.0445	0.0344	0.0503

Table 13. Pole Position Errors (sec of arc) From Equation (3) Plus Semiannual Terms; Data From 1975

Chandler Period (Days)	Number of Points in fit	Pole Component	Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
			rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
445	87	X	0.0095	0.0198	0.0121	0.0275	0.0147	0.0343	0.0175	0.0405
		Y	0.0047	-0.0099	0.0061	-0.0131	0.0076	0.0169	0.0092	0.0206
	85	X	0.0094	0.0211	0.0122	0.0291	0.0149	0.0363	0.0179	0.0428
		Y	0.0041	-0.0091	0.0055	-0.0121	0.0070	0.0157	0.0085	0.0192
435	89	X	0.0095	0.0192	0.0119	0.0236	0.0142	0.0297	0.0167	0.0349
		Y	0.0050	0.0106	0.0063	-0.0132	0.0077	0.0159	0.0091	0.0193
	87	X	0.0095	0.0192	0.0120	0.0226	0.0145	0.0332	0.0172	0.0390
		Y	0.0045	-0.0097	0.0059	-0.0127	0.0073	0.0156	0.0087	0.0190
425	89	X	0.0094	0.0206	0.0121	0.0284	0.0147	0.0353	0.0177	0.0415
		Y	0.0040	-0.0089	0.0053	-0.0177	0.0068	0.0146	0.0082	0.0178
	87	X	0.0094	0.0184	0.0118	0.0226	0.0140	0.0283	0.0164	0.0332
		Y	0.0049	0.0098	0.0061	0.0128	0.0073	0.0143	0.0087	0.0174
	87	X	0.0093	0.0179	0.0118	0.0247	0.0142	0.0307	0.0168	0.0359
		Y	0.0044	0.0092	0.0056	0.0120	0.0069	0.0129	0.0082	0.0155
	85	X	0.0092	0.0194	0.0119	0.0267	0.0144	0.0331	0.0173	0.0387
		Y	0.0039	-0.0084	0.0051	-0.0109	0.0064	0.0122	0.0078	0.0148



Table 14. Pole Position Prediction Errors (sec of arc) From Equation (3)  
Plus Semiannual Terms; Data From 1976

Chandler Period (Days)	Number of Points in fit	Pole Component	Five-Day Predictions		10-Day Predictions		15-Day Predictions		20-Day Predictions	
			rms	Maximum	rms	Maximum	rms	Maximum	rms	Maximum
445	87	X	0.0113	0.0182	0.0136	0.0248	0.0173	0.0314	0.0202	0.0380
		Y	0.0050	-0.0094	0.0062	-0.0122	0.0077	-0.0156	0.0091	-0.0176
	85	X	0.0111	0.0178	0.0135	0.0243	0.0176	0.0308	0.0209	0.0372
		Y	0.0050	0.0100	0.0064	-0.0129	0.0079	-0.0165	0.0093	-0.0187
435	89	X	0.0117	0.0196	0.0138	0.0239	0.0172	0.0304	0.0199	0.0369
		Y	0.0052	-0.0099	0.0065	-0.0129	0.0080	-0.0166	0.0094	-0.0189
	87	X	0.0114	0.0184	0.0137	0.0251	0.0175	0.0318	0.0205	0.0385
		Y	0.0051	-0.0099	0.0064	-0.0129	0.0079	-0.0166	0.0093	-0.0188
	85	X	0.0111	0.0180	0.0136	0.0246	0.0178	0.0311	0.0211	0.0377
		Y	0.0051	-0.0104	0.0065	-0.0136	0.0080	-0.0174	0.0095	-0.0198
425	89	X	0.0119	0.0200	0.0141	0.0242	0.0175	0.0308	0.0203	0.0374
		Y	0.0054	-0.0107	0.0068	-0.0139	0.0084	-0.0178	0.0099	-0.0203
	87	X	0.0116	0.0186	0.0139	0.0254	0.0178	0.0322	0.0209	0.0389
		Y	0.0052	-0.0105	0.0066	-0.0137	0.0082	-0.0176	0.0097	-0.0201
	85	X	0.0113	0.0182	0.0137	0.0248	0.0180	0.0315	0.0213	0.0381
		Y	0.0052	-0.0110	0.0066	-0.0143	0.0082	-0.0182	0.0098	-0.0209

### SUMMARY

Various methods and equations were tested for the prediction of UT1-UTC, the effects of variations in the earth's rate of rotation, and the position of the instantaneous pole. A reasonable choice for a UT1-UTC prediction model, considering both prediction accuracy and model simplicity, is a least-squares fit using one year of data to determine a constant, linear coefficient and coefficients of a trigonometric series with periods of 91, 121, 182, and 365 days (Equation (1)). For this model, the rms of the prediction errors for each year from 1973 through 1975 was 0.002 to 0.003 sec of time for a 5-day prediction span, and grew to 0.003 to 0.007 sec for 20-day prediction spans. These prediction errors are at least 30 percent smaller than the linear fit made from two data points.

The optimum model was different for the X and Y components of pole position for unknown reasons. A fit of a constant and the Chandler and annual coefficients (Equation (3)) to 435 days (the Chandler Period) of the X component of pole position resulted in rms errors for 1975 and 1976 of 0.008 to 0.010 sec of arc for a 5-day prediction and about 0.012 sec of arc for a 20-day prediction. For the Y component, the addition of semiannual terms to the parameter set yielded rms errors of 0.005 sec of arc for the 5-day predictions and 0.009 sec for the 20-day prediction. These errors are nearly 50 percent less than the errors obtained by a fit of a constant and Chandler coefficients (Equation (2)) to 200 days of data. All fits were made to data determined by the Bureau International De L'Heure. It may be of interest to conduct a test using IPMS and Doppler data to determine if different parameter sets are required for X and Y components of pole positions determined by other data reduction and observing techniques.

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